ISSN 0798 1015

REVISTA ESPACIOS

EDUCACIÓN • EDUCAÇÃO • EDUCATION

Vol. 40 (Number 29) Year 2019. Page 8

Unified method for teaching how to solve the equivalent circuit of transformers

Método unificado para enseñar a resolver el circuito equivalente de transformadores

ORTIZ-CASTRILLÓN, José R. 1; JARAMILLO-DUQUE, Álvaro 2; MUÑOZ-GALEANO, Nicolás 3; SIERRA-AGUILAR, Juan E. 4 & LOPEZ-LEZAMA, Jesús M. 5

Received: 12/04/2019 • Approved: 13/08/2019 • Published 02/09/2019

Contents

ABSTRACT:

In this paper, a unified method for calculating variables and parameters of transformers considering power losses in their windings is proposed. This method presents an educational tool for teaching how to obtain the steady-state operating conditions of transformers used to make technical and economic decisions. Five variants are considered in calculations using single-phase, three-phase or per-unit (p.u) equations. This method is validated by calculations and simulations (ATPDraw); also, an on-line PYTHON graphic interface was implemented.

Keywords: Transformers, solution method, simplified model, education

RESUMEN:

En este artículo se propone un método unificado para calcular las variables y parámetros de transformadores considerando pérdidas de potencia en sus devanados. Este método presenta una herramienta educativa para enseñar a obtener las condiciones operativas de transformadores en estado-estable usadas para tomar decisiones técnicas y económicas. Se consideran cinco variantes para los cálculos usando ecuaciones monofásicas, trifásicas y por-unidad (p.u). Este método se valida con cálculos y simulaciones (ATPDraw); además, se implementó una interfaz gráfica on-line en PYTHON.

Palabras clave: Transformadores, método de solución, modelo simplificado, educación

1. Introduction

Transformers in electrical systems are used as a link between generation and transmission, as well as transmission and distribution, and distribution and final users. The main function of transformers in transmission systems consist on increasing the voltage magnitude for reducing electric losses in the lines. In distribution systems, transformers are commonly used for decreasing voltage magnitude up to operation levels and feeding electric devices minimizing security risks for people. In addition, transformers have windings with galvanic isolation and power transference is given by means of electromagnetic induction, providing isolation between electrical systems; in consequence, the transformer protection is essential for electrical grids (Vahidi & Esmaeeli, 2013), (Varan & Yurtsever, 2017). Transformers are also used for improving power quality, controlling angle-phase (Thompson, Miller, & Burger, 2008), starting motors (Huan, Xiangrui, & Guangzhe, 2012), and filtering third order and zero sequence harmonics (Ionescu, Paltanea, & Paltanea, 2013), (Hurng-Liahng, Kuen-Der, Jinn-Chang, & Wen-Jung, 2008).

Network operators need to know transformers operating conditions due to technical and economic reasons (Xiao-pin, Yi-yi, Yue, & Bin, 2014), (Georgilakis & Amoiralis, 2010). Transformer variables such as feeding voltage, output voltage, currents and load data are used in reliability studies, overload voltage regulation, disconnections and circuit transferences (Yazdani-Asrami, Mirzaie, & Shayegani Akmal, 2010). Hence, electrical engineers and students of electrical engineering need a methodology for learning how to find the transformer variables and parameters.

Generally, the conventional model is used for studying the transformer behavior in steady state (Jiale, Jiao, Song, & Kang, 2009). The conventional model is based on the transformer equivalent circuit where its windings are represented as inductors with resistances connected in series and the transformer core is represented as an inductor with a resistance connected in parallel. Nevertheless, the conventional model is modified according to the study to be done, for example: the high frequency model is formed adding to the conventional model the capacitive couplings between its windings. This model is used for designing transformers for isolation and voltage impulse tests (Nagy & Osama, 2010). The conventional model is combined with the transformer magnetic model for analyzing inrush currents in the magnetization branch (Abdulsalam, Xu, & Dinavahi, 2005). On the other hand, the conventional model is adapted to the RLC model when it is necessary to study transformer responses due to transient states (Hassan Hosseini, Vakilian, & Gharehpetian, 2008), (Yang, Wang, Cai, & Wang, 2011), (Celis-Montero, Castro-Aranda, & Martinez-Velasco, 2012), (Vahidi, Agheli, & Jazebi, 2012).

In this paper, the transformer solution method is deduced in steady state from a simplified model based on the conventional model. The main contribution of this paper consists on providing an educational tool for students of electrical engineering and electrical engineers regarding the solution of the transformer simplified model. The proposed solution allows obtaining voltages, currents, powers, power losses, efficiency, voltage regulation and power factor by means of nominal characteristics (transformer identification plate), test results (Ayasun & Nwankpa, 2006) and load connected. The proposed method has five variants: 1) single-phase solved from transformer primary side, 2) single-phase solved from transformer secondary side, 3) three-phase solved from transformer primary side, 4) three-phase solved from transformer secondary side and 5) per unit solution. Each variant can be applied according to the windings connection (wye or delta).

This paper is organized as follows: Section 1 presents the transformer conventional model based on power losses and the use of the simplified model is justified; in section 2, the solution method with its five variants is described; Section 3 presents the method validation through mathematical calculations and simulation in ATPDraw; in Section 4, the graphic interface for solving three-phase transformers is shown; finally, in section, 5 the most relevant conclusions and discussions are presented.

---

2. Methodology

2.1. Transformer ideal model

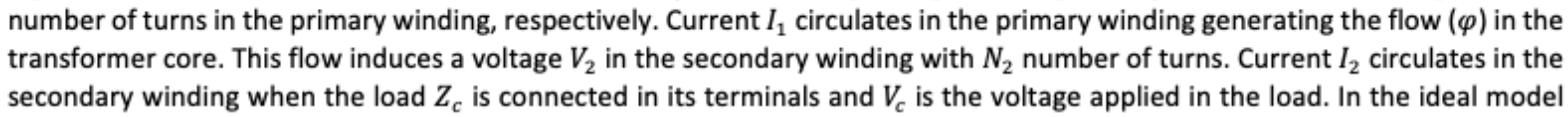
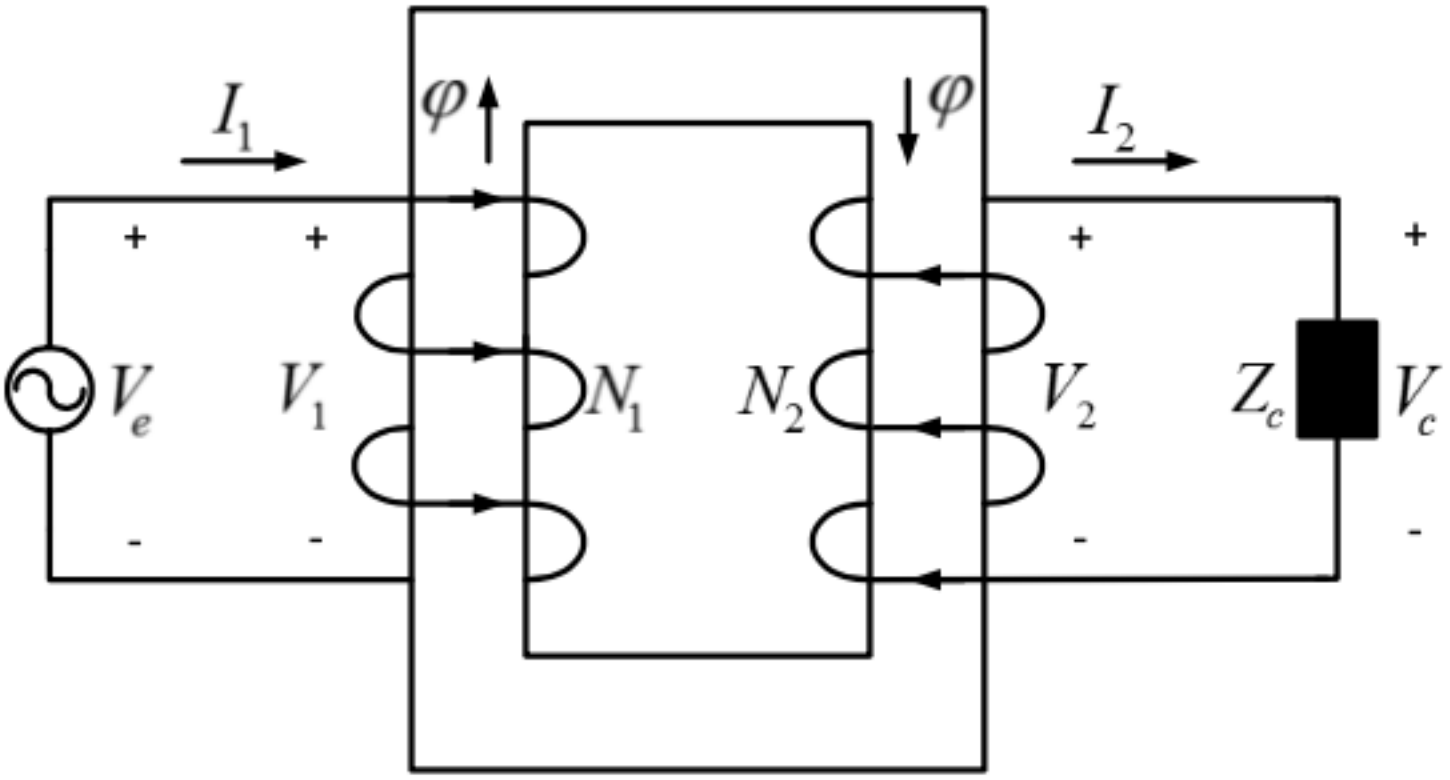


Figure 1 shows the transformer's ideal model where V_e is the feeding voltage while V_1 and N_1 are the induced voltage and the number of turns in the primary winding, respectively. Current I_1 circulates in the primary winding generating the flow (φ) in the transformer core. This flow induces a voltage V_2 in the secondary winding with N_2 number of turns. Current I_2 circulates in the secondary winding when the load Z_c is connected in its terminals and V_c is the voltage applied in the load. In the ideal model $V_e = V_1$ and $V_2 = V_c$; that is because power losses are not considered in the transformer windings.

Figure 1
Transformer's
ideal model

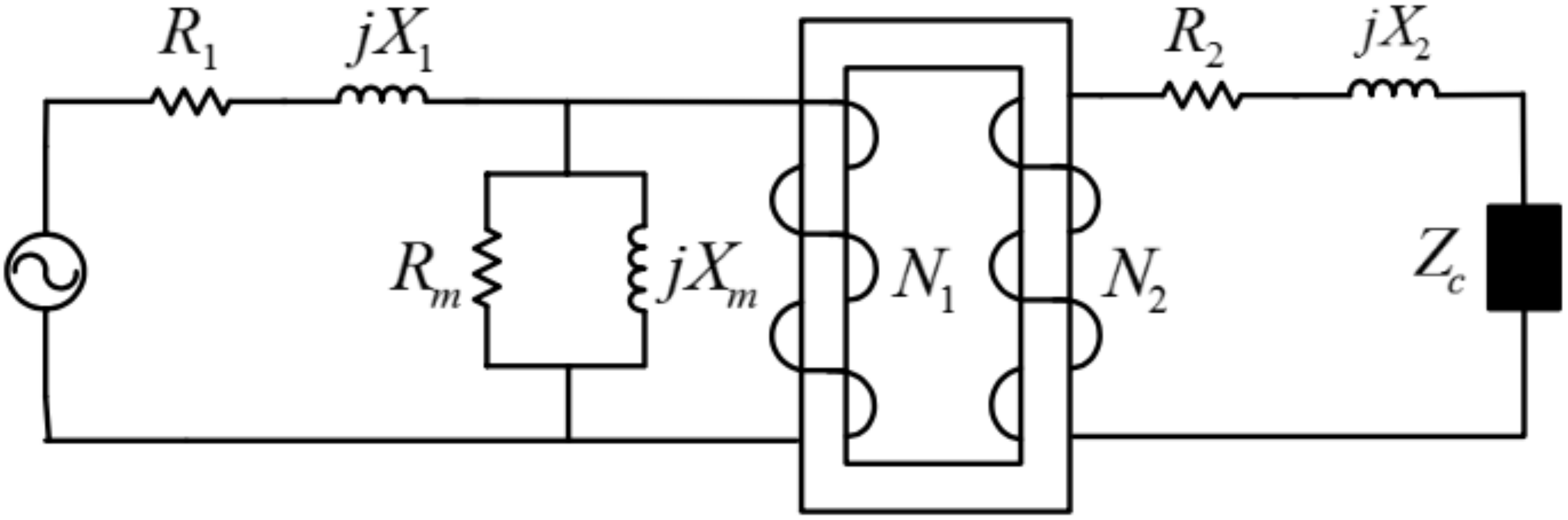


The ideal model is not suitable for studying transformers operation in steady state; in this model, it is considered that all power from the feeding source is given to the load; in consequence, the dispersion flows as well as power losses are not considered. This model does not take into account the power dissipated by the transformer as heat (power losses by Joule effect), parasite currents (power losses by Foucault effect) and core magnetization (power losses by hysteresis effect).

2.2. Transformer model based on losses

Figure 2 shows the transformer equivalent circuit that models power losses as resistances (Martínez-Velasco & de León, 2011). The primary winding impedance is represented as the inductor X_1 and the conductor losses as R_1. The magnetization branch is represented as the inductor X_m and core losses as R_m. The secondary winding impedance is represented as the inductor X_2 and the conductor losses as R_2. Inductors X_1, X_2 also model the dispersion flows in the primary and secondary windings, respectively. The inductor X_m models excitation current effects in the transformer core.

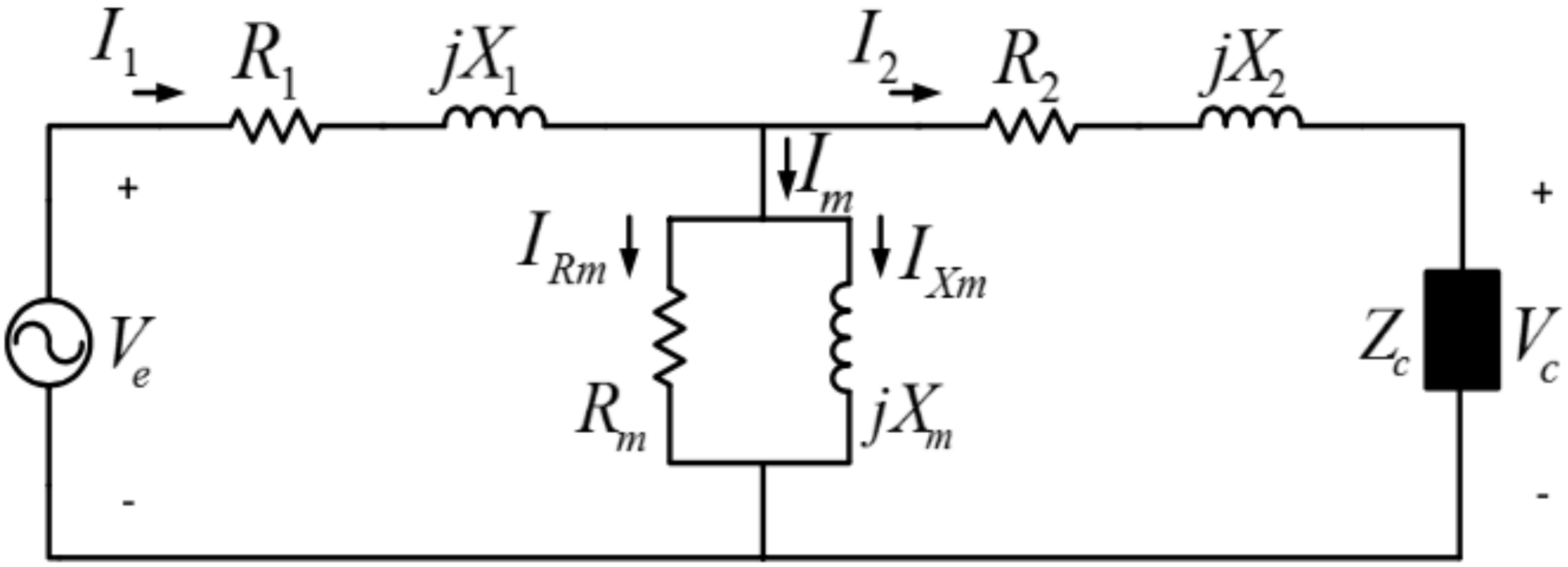
Figure 2
Transformer model
based on losses



The transformer model depicted in figure 2 can be simplified as follows: the impedances of the secondary side are transferred to the primary side (model referred to primary side); or conversely, the impedances of the primary side are transferred to the secondary side (model referred to secondary side). Figure 3 shows the transformer equivalent circuit based on losses or transformer conventional model which is used when calculations are referred to one transformer side.

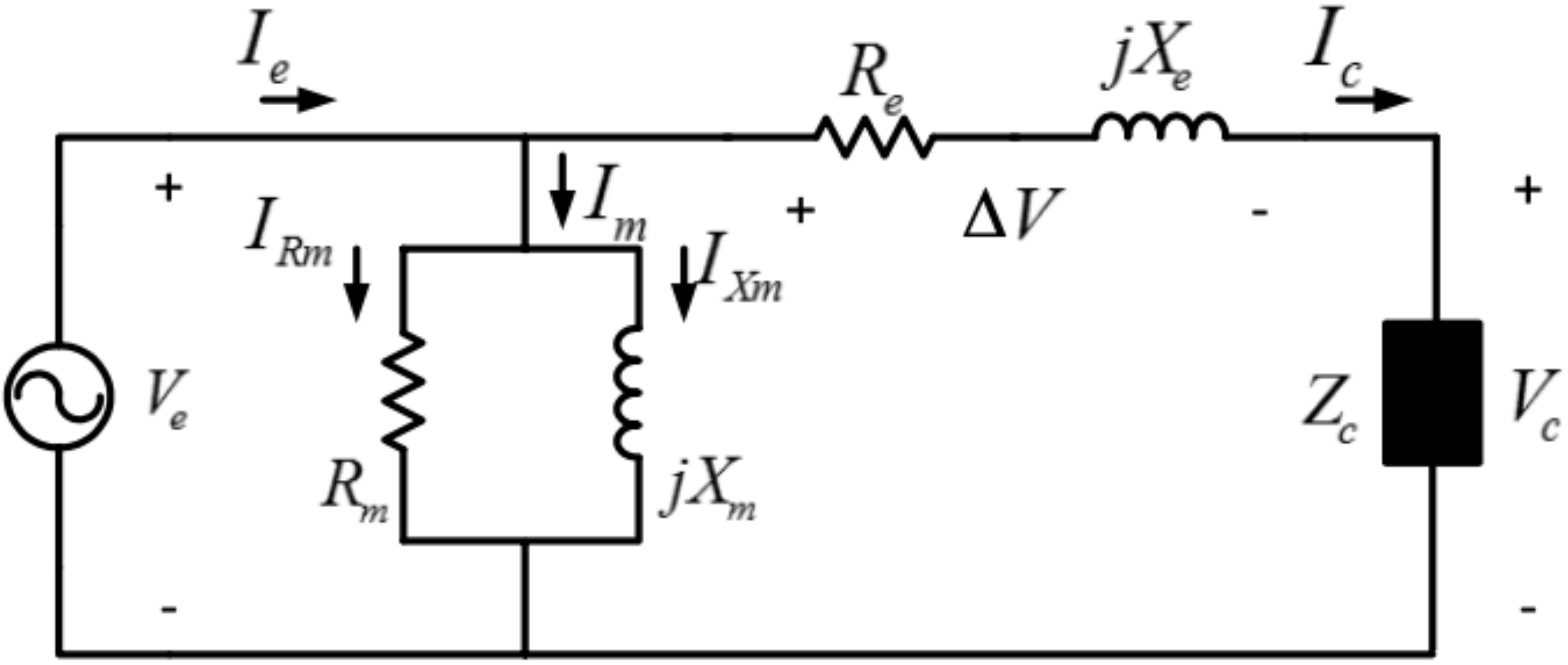
Figure 3
Transformer conventional

model.



In figure 3, the magnetization impedance formed by the parallel between R_m and jX_m ($Z_m = R_m || jX_m$) can be moved to the primary winding terminals. Thus, windings impedances are in series and they can be added to obtain the transformer simplified model depicted in figure 4.

Figure 4
Transformer
simplified model



The transformer's simplified model can be used without introducing significant mistakes in calculations because core losses depend on the feeding voltage magnitude. The voltage magnitude does not significantly change because voltage drop in windings is usually small compared with the feeding voltage. In the transformer simplified model, the primary and secondary windings are represented with their series equivalent circuit formed by $R_e + jX_e$ where: $R_e = R_1 + R_2'$ and $X_e = X_1 + X_2'$, this when it is necessary to solve the transformer referred to the primary side. $R_e = R_1' + R_2$ y $X_e = X_1' + X_2$ are used when it is necessary to solve the transformer referred to the secondary side. Primed variables correspond to resistance and inductance of primary and secondary sides referred to the secondary and primary sides, respectively.

In this paper, the simplified model of figure 4 is used for analyzing the equivalent circuit solution. This model is used due to the fact that transformers can be adequately modeled without introducing significant mistakes in the calculations. Additionally, the equivalent circuit solution of the simplified model is simpler than the solution by means of the models illustrated in figures 2 and 3.2.3. Unified solution method

In this section, the proposed method for solving transformer equations is explained. The method has 5 variants according to transformer vector group and reference side for solving the equivalent circuit. The method variants are: 1) single-phase referred to secondary side (SS), 2) three-phase referred to secondary side (TS), 3) single-phase referred to primary side (SP), 4) three-phase referred to primary side (TP) and 5) the per unit (p.u) system.

In variants SS and SP voltages, currents and powers per phase are used, and results are given in per phase values. In variants TS and TP three-phase powers with line-line voltages and line currents are used; results are also given in three-phase values. In variant 5 (per unit) all variables and parameters are changed to p.u system and results are also in p.u system.

The solution method allows solving the transformer equivalent circuit independently of the selected variant. Coefficients (k_i, $i = 1,2,\ldots.8$) are used to simplify the solution of equations since they are similar for all variants. Coefficients are selected according to the chosen variant and transformer vector group. Coefficients are explained at end of this section.

2.4. General definitions for transformer variables and parameters

In this section, the nomenclature is explained. Parameters and variables in bold style refer to a phasor quantity. Parameters and variables with no bold style correspond to magnitudes. Parameters and variables with only one sub-index correspond to per unit system or test data.

Transformer parameters are: $\boldsymbol{Z}_{*\_\#}$: impedance, $R_{*\_\#}$: resistance and $X_{*\_\#}$: inductance. transformer variables are: $\boldsymbol{S}_{*\_\#}$: apparent power, $P_{*\_\#}$: active power, $V_{*\_\#}$: voltage, $\Delta\boldsymbol{V}_{*\_\#}$: voltage drop, $Vreg$: voltage regulation, $I_{*\_\#}$: current, a: voltage ratio, fp: power factor, and η: efficiency.

Sub-indexes " $*$ " and "#" are used for simplifying the parameters and variables writing. Sub-index " $*$ " indicates the parameter or variable type ($*= 1,3,b,eq,m,pu,ll,ln,i,fe,cu,fd$), where 1: single-phase, 3: three-phase, b: base, eq: equivalent, m: magnetization, pu: per unit, ll: line-line, ln: line-neutral, i: input, cu: copper losses, fe: iron losses, fd: magnetization. Sub-index "#" indicates the reference side of parameter or variable ($\# = t,p,s,n,c$) where t: transformer, p: primary, s: secondary, n: nominal and c: load. Thus, table 1 shows nomenclature of parameters and variables according to method variant. For example, the variant TS has the base impedance $Z_{b\_\#} = Z_{b\_s}$, being the sub index s to indicate that the impedance is in transformer secondary side.

Table 1
Parameters and variables according to variant selected

variant Parameter or variable	SS	TS	SP	TP	p.u
$Z_{b\_\#}$	Z_{b_s}	Z_{b_s}	Z_{b_p}	Z_{b_p}	Z_b
$\boldsymbol{Z}_{eq\_\#}$	$\boldsymbol{Z}_{eq\_s}$	$\boldsymbol{Z}_{eq\_s}$	$\boldsymbol{Z}_{eq\_p}$	$\boldsymbol{Z}_{eq\_p}$	$\boldsymbol{Z}_{eq}$
V_{*_c}	V_{ln_c}	V_{ll_c}	V_{ln_c}	V_{ll_c}	V_c
S_{*_c}	S_{1_c}	S_{3_c}	S_{1_c}	S_{3_c}	S_c
$\boldsymbol{I}_{*\_c}$	$\boldsymbol{I}_{f\_c}$	$\boldsymbol{I}_{l\_c}$	$\boldsymbol{I}_{f\_c}$	$\boldsymbol{I}_{l\_c}$	$\boldsymbol{I}_c$
$\Delta\boldsymbol{V}_{*\_\#}$	$\Delta\boldsymbol{V}_{ln\_t}$	$\Delta\boldsymbol{V}_{ll\_t}$	$\Delta\boldsymbol{V}_{ln\_t}$	$\Delta\boldsymbol{V}_{ll\_t}$	$\Delta\boldsymbol{V}_t$
$\boldsymbol{V}_{*\_e}$	$\boldsymbol{V}_{ln\_e}$	$\boldsymbol{V}_{ll\_e}$	$\boldsymbol{V}_{ln\_e}$	$\boldsymbol{V}_{ll\_e}$	$\boldsymbol{V}_e$
$R_{m\_\#}$	R_{m_s}	R_{m_s}	R_{m_p}	R_{m_p}	R_m
$\boldsymbol{X}_{m\_\#}$	$\boldsymbol{X}_{m\_s}$	$\boldsymbol{X}_{m\_s}$	$\boldsymbol{X}_{m\_p}$	$\boldsymbol{X}_{m\_p}$	$\boldsymbol{X}_m$
$\boldsymbol{Z}_{m\_\#}$	$\boldsymbol{Z}_{m\_s}$	$\boldsymbol{Z}_{m\_s}$	$\boldsymbol{Z}_{m\_p}$	$\boldsymbol{Z}_{m\_p}$	$\boldsymbol{Z}_m$
$\boldsymbol{I}_{m\_\#}$	$\boldsymbol{I}_{m\_s}$	$\boldsymbol{I}_{m\_s}$	$\boldsymbol{I}_{m\_p}$	$\boldsymbol{I}_{m\_p}$	$\boldsymbol{I}_m$
$\boldsymbol{I}_{*\_e}$	$\boldsymbol{I}_{f\_e}$	$\boldsymbol{I}_{l\_e}$	$\boldsymbol{I}_{f\_e}$	$\boldsymbol{I}_{l\_e}$	$\boldsymbol{I}_e$

2.5. Input data

It is considered that in the transformer secondary side a load is connected with the following characteristics: S_{3_c} is three-phase power, V_{ll_c} is line-line voltage and power factor fp in lagging (↓) if load is resistive-inductive or leading (↑) if load is resistive-capacitive. Transformer nominal conditions (from the identification plate) are: three-phase power (S_{3_n}); line-line primary voltage (V_{ll_p}); line-line secondary voltage (V_{ll_s}); short-circuit impedance or equivalent impedance in p.u ($\boldsymbol{Z}_{cc} = Z_{cc}\measuredangle\theta_{cc}$, where θ_{cc} is the impedance angle); iron power losses in p.u (P_{fe}); excitation power in p.u (Q_{fd}) and finally transformer windings connection (YY=wye-wye, ΔY=delta-wye, $\Delta\Delta$=delta-delta, $Y\Delta$=wye-delta). On the other hand, the transformer test data can also be used to calculate the transformer impedances. 1) Short-circuit test: V_{sh}, I_{sh} and P_{sh} are short-circuit voltage, short-circuit current and short-circuit power, respectively. 2) Open-circuit test: V_{oc}, I_{oc} and P_{oc} are open-circuit voltage, open-circuit current and open-circuit power, respectively.

2.6. Proposed solution method

The general solution, step by step is explained as follows:

1) Nominal voltage ratio a_n is calculated according to equation 1.

$$a_n = \frac{V_{ll\_p}}{V_{ll\_s}} \tag{1}$$

Then, the method variant is chosen (SS, TS, SP, TP or p.u). Voltage ratio a is selected in table 2 according to the method variant and windings connection. This voltage ratio is used to change the voltages at the reference side when it is necessary.

Table 2
Voltage ratio *a*

Windings connection \ Variant	MS	TS	MP	TP	p.u
YY	1	1	a_n	a_n	1
ΔY	1	1	$\sqrt{3} \cdot a_n$	a_n	1
$\Delta\Delta$	1	1	a_n	a_n	1
$Y\Delta$	1	1	$\frac{1}{\sqrt{3}} \cdot a_n$	a_n	1

In variants SS and TS, $a = 1$ for all winding connections. That is because the load is connected in the secondary side (parameters and variables are in their reference side). In variant p.u, $a = 1$ for all winding connections. In variant TP, $a = a_n$ for all winding connections since it is necessary to change the load reference to primary side from secondary side. In variant SP, $a = a_n$ for YY and $\Delta\Delta$. That is because it is necessary to change the reference from the secondary side to the primary side. In connection ΔY, a_n is multiplied by $\sqrt{3}$ because it is necessary to change V_{ll_s} of equation 1 to voltage per phase (V_{ln_s}) (corresponding to wye connection). In connection $Y\Delta$, a_n is divided by $\sqrt{3}$ because it is necessary to change V_{ll_p} of equation (1) to voltage per phase (V_{ln_p}) (corresponding to wye connection).

2) The base impedance is calculated according to equation (2).

3) The equivalent impedance phasor is obtained as follows: equation (3a) is used when $\mathbf{Z}_{eq\_\#}$ is calculated with the nominal short-circuit impedance in p.u (identification plate data) and base impedance; equation 3b is used when $\mathbf{Z}_{eq\_\#}$ is calculated with transformer test data.

4) The load voltage is calculated according to the winding connection and reference side.

5) The apparent power magnitude in the load is calculated (equation 5) according to the selected variant.

6) The load current phasor is calculated (equation 6). The current angle θ_{Ic} is obtained from the load power factor; θ_{Ic} is positive if the load is resistive-capacitive or negative if it is inductive-resistive.

7) The voltage drop due to the transformer windings is calculated (equation 7).

8) The input voltage is calculated (equation 8) as the sum of the load voltage and transformer voltage drop.

9) The magnetization resistance is calculated as follows: equation 9*a* is used to calculate $R_{m\_\#}$ with transformer nominal values; equation 9*b* is used when $R_{m\_\#}$ is calculated using the open-circuit test data and θ_m is the magnetization impedance angle.

10) The magnetization inductance is calculated as follows: equation 10*a* is used to calculate $\mathbf{X}_{m\_\#}$ with transformer nominal values; equation 10*b* is used when $\mathbf{X}_{m\_\#}$ is calculated using the open-circuit test data.

11) The magnetization impedance is calculated (equation 11) as the parallel between the magnetization resistance and the magnetization inductance.

12) The magnetization current is calculated (equation 12) as the relation between the input voltage and the magnetization impedance.

13) The input current is calculated (equation 13) adding the load current and the magnetization current.

14) The transformer iron losses are calculated (equation 14) using the input voltage and the magnetization resistance.

15) The transformer copper losses are calculated (equation 15) with the load current and the real part (resistance) of the equivalent impedance.

16) The transformer efficiency is calculated (equation 16) as the relation between output power and input power. In this step, the load power factor and transformer losses are considered.

17) Voltage regulation percentage is calculated (equation 17) using load voltage and transformer input voltage.

18) Input power factor is calculated (equation 18) with the input voltage angle θ_{Ve} and the input current angle θ_{Ie} of the transformer.

$$Z_{b\_\#} = \frac{(V_{ll\_s})^2}{S_{3\_n}} \tag{2}$$

$$\mathbf{Z}_{eq\_\#} = k_1 \cdot Z_{b\_\#} \cdot \mathbf{Z}_{eq} \tag{3a}$$

$$\mathbf{Z}_{eq\#} = k_1 \cdot \frac{V_{sh}}{I_{sh}} \measuredangle \theta_{eq} \;\therefore\; \theta_{eq} = \cos^{-1}\left(\frac{P_{sh}}{V_{sh} \cdot I_{sh}}\right) \tag{3b}$$

$$V_{*\_c} = k_2 \cdot V_{ll\_c} \tag{4}$$

$$S_{*\_c} = k_3 \cdot S_{3\_c} \tag{5}$$

$$\boldsymbol{I}_{*\_c} = k_4 \cdot \frac{S_{*\_c}}{V_{*\_c}} \measuredangle \theta_{Ic} \;\therefore\; \theta_{Ic} = \pm \cos^{-1}(fp) \tag{6}$$

$$\Delta \boldsymbol{V}_{*\_\#} = k_5 \cdot \boldsymbol{Z}_{eq\_\#} \cdot \boldsymbol{I}_{*\_c} \tag{7}$$

$$\boldsymbol{V}_{e\_\#} = \Delta \boldsymbol{V}_{*\_\#} + \boldsymbol{V}_{*\_c} \tag{8}$$

$$R_{m\_\#} = k_1 \cdot \frac{V_{ll\_s}^2}{P_{fe\_n} \cdot S_{3\_n}} \tag{9a}$$

$$R_{m\#} = k_1 \cdot \frac{V_{oc}}{I_{oc} \cdot \cos(\theta_m)} \;\therefore\; \theta_m = \cos^{-1}\left(\frac{P_{oc}}{V_{oc} \cdot I_{oc}}\right) \tag{9b}$$

$$\boldsymbol{X}_{m\_\#} = k_1 \cdot \frac{V_{ll\_s}^2}{Q_{fd\_n} \cdot S_{3\_n}} \measuredangle 90^o \tag{10a}$$

$$\boldsymbol{X}_{m\_\#} = k_1 \cdot \frac{V_{oc}}{I_{oc} \cdot \sin(\theta_m)} \measuredangle 90^o \tag{10b}$$

$$\boldsymbol{Z}_{m\_\#} = \frac{R_{m\_\#} \cdot \boldsymbol{X}_{m\_\#}}{R_{m\_\#} + \boldsymbol{X}_{m\_\#}} \tag{11}$$

$$\boldsymbol{I}_{m\_\#} = k_6 \cdot \frac{\boldsymbol{V}_{e\_\#}}{\boldsymbol{Z}_{m\_\#}} \tag{12}$$

$$\boldsymbol{I}_{e\_\#} = \boldsymbol{I}_{m\_\#} + \boldsymbol{I}_{*\_c} \tag{13}$$

$$P_{fe\_t} = k_7 \cdot \frac{V_{e\_\#}^2}{R_{m\_\#}} \tag{14}$$

$$P_{cu\_t} = k_8 \cdot I_{*\_c}^2 \cdot Z_{eq\_\#} \cdot \cos\left(\theta_{eq}\right) \tag{15}$$

$$\eta = \frac{S_{*\_c} \cdot fp \cdot 100\%}{S_{*\_c} \cdot fp + P_{fe\_t} + P_{cu\_t}} \tag{16}$$

$$Vreg = \frac{V_{e\_\#} - V_{*\_c}}{V_{*\_c}} \cdot 100\% \tag{17}$$

$$fp_e = \cos(\theta_{Ve} \pm \theta_{Ie}) \tag{18}$$

Table 3 shows the values of k_i according to the variant selected and windings connection. For example, if a transformer with windings connection ΔY and variant SP is selected, then, the k_i constants of column Δ are used because the primary winding connection is Δ. Additionally, equations (2) (9) and (10) are not altered when $k_i = 1$.

Constant k_1 is used to multiply the impedances in variants SP and TP by a^2; this is done to transfer the impedances at the primary side. In delta connections, k_1 is used to multiply by 3 according to the theorem of Kennelly for wye-delta transformations (impedance in delta is equal to three times wye impedance). In the p.u variant, this constant is used to divide the impedances by the base impedance; on the other hand, equation 3a has $k_1 = 1$ in p.u variant because the short-circuit impedance is taken from the identification plate (equation 3a does not need a change in this case).

Constant k_2 is used to change the line-line voltage to phase voltage if variant are SP or SS; similarly, the constant is used to change line-neutral voltage to line-line voltage in three-phase variants (TP or TS). Also, constant uses voltage ratio a_n to change the reference side from secondary side to primary side for variants referred to transformer primary side.

Constant k_3 divides by 3 in single-phase variants to transform the three-phase power into power per phase.

Constant k_4 divides by $\sqrt{3}$ in three-phase variants for obtaining the line current.

Constant k_5 divides by $\sqrt{3}$ in delta connections for changing the line current into phase current; also, this constant multiplies by $\sqrt{3}$ in wye connections for changing the phase-neutral voltage into line-line voltage.

Constant k_6 divides by $\sqrt{3}$ in wye connections for changing line-line voltage into phase-neutral voltage; also, this constant multiplies by $\sqrt{3}$ in delta connections for changing phase current into line current.

Constant k_7 divides by $\sqrt{3}$ in wye connections for changing line-line voltage into line-neutral voltage; also, this constant multiplies by 3 in three-phase variants for changing the phase power into three-phase power.

Constant k_8 divides by $\sqrt{3}$ in delta connections for changing the line current into phase current; also, this constant multiplies by 3in three-phase variants for changing the phase power into three-phase power.

Table 3
Constants k_i according to method variant

Variant	**Reference: secondary side**				**Reference: primary side**				**p.u**
	SS		**TS**		**SP**		**TP**		
Constant	Y	Δ	Y	Δ	Y	Δ	Y	Δ	
k_1	1	3	1	3	a_n^2	$3 \cdot a_n^2$	a_n^2	$3 \cdot a_n^2$	$\frac{1}{Z_{b\_\#}}$
k_2	$\frac{1}{\sqrt{3}}$	1	1	1	$\frac{a}{\sqrt{3}}$	a	a	a	$\frac{1}{V_b}$
k_3	$\frac{1}{3}$	$\frac{1}{3}$	1	1	$\frac{1}{3}$	$\frac{1}{3}$	1	1	$\frac{1}{S_b}$
k_4	1	1	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	1	1	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	1
k_5	1	1	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	1	1	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	1
k_6	1	1	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1	1	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1
k_7	1	1	$\frac{3}{(\sqrt{3})^2}$	3	1	1	$\frac{3}{(\sqrt{3})^2}$	3	1
k_8	1	1	3	$\frac{3}{(\sqrt{3})^2}$	1	1	3	$\frac{3}{(\sqrt{3})^2}$	1

3. Results

The proposed method is validated in ATPDraw software for a transformer with the following characteristics or identification plate:

$S_{3\_n} = 500\ kVA, V_{ll\_p} = 44\ kV, V_{ll\_s} = 13.2\ kV$
$\boldsymbol{Z_e} = 0.08\measuredangle 80^o, P_{fe\_n} = 0.03, Q_{fd\_n} = 0.02$. Connection=$\Delta Y11$.

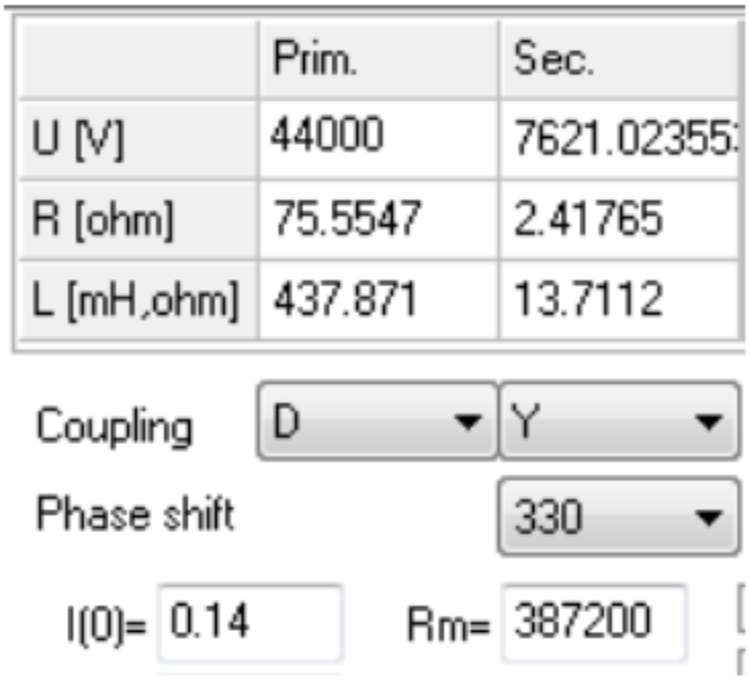
Load characteristics:
$S_{3\_c} = 400\ kVA, V_{ll\_c} = 13.1\ kV, fp = 0.9 \downarrow$.

The validation of the proposed approach was performed using theoretical calculations for all variants; the results are compared with ATPDraw results. In this case S_{3_n} and V_{ll_s} are used as base values in variant p.u.
Figure 5 illustrates the simulated circuit in ATPDraw where the transformer is supplied by a three-phase voltage source; the transformer has a load in wye connection. The resistance and inductor are used to model a load with lagging power factor.

Figure 5
Circuit to simulate
in ATPDraw

ATPDraw uses the transformer model of figure 2 for simulation; hence, it is necessary to change the model of the proposed method (figure 4) to the model depicted in figure 2 for simulation purposes. Figure 6 shows the transformer's input data where the primary and secondary rated voltages are rms magnitudes per phase. The resistance and inductance of the primary side correspond to $Z_1 = R_1 + jX_1$ of figure 2. Resistance and inductance of the secondary side correspond to $Z_2 = R_2 + jX_2$ of figure 2. Then, Z_1 and Z_2 correspond to the equivalent impedance of figure 4 ($Z_{eq\_p\_} = Z_1 + Z_2'$). Vector group $\Delta Y11$ causes a phase shift of 330 degrees. Magnetization resistance is calculated using equation 6 of the SP variant. Finally, magnetization current is calculated using equation 12 of the SP variant; ATPDraw calculates the magnetization inductance using the magnetization current in the simulation process.

Figure 6
Input data for
ATPDraw simulation



	Prim.	Sec.
U [V]	44000	7621.02355
R [ohm]	75.5547	2.41765
L [mH,ohm]	437.871	13.7112

Coupling D Y

Phase shift 330

I(0)= 0.14

Rm= 387200

Table 4 shows the calculated results for each variant and ATPDraw simulations results. In this case, the label NA indicates that it is not possible to obtain the corresponding parameter or variable from the ATPDraw simulation.

Simulation results in ATPDraw are validated as follows: 1) the calculation results from variant SP are validating with the variables and parameters measured in transformer primary side from ATPDraw; 2) the variant SS calculations are validated by means of variables and parameters that are measured in the transformer secondary side from ATPDraw. Results are presented in table 4 where each row has two values in bold, the calculated value by the proposed method and its corresponding value from the ATPDraw simulation.

Table 4
Results of
proposed method

Variant **Step**	**SS**	**TS**	**SP**	**TP**	**p.u**	**ATPDraw**
(1) a_n	3.33	3.33	3.33	3.33	1	3.33
(1) a	1	1	**5.77**	3.33	1	**5.77**
(2) $Z_{b\_\#}$ [Ω]	348.48	348.48	11616	11616	NA	NA
(3a) $\boldsymbol{Z}_{e\_\#}$ [Ω]	$27.88\measuredangle 80^o$	$27.88\measuredangle 80^o$	$929.28\measuredangle 80^o$	$929.28\measuredangle 80^o$	$0.08\measuredangle 80^o$	NA
(4) V_{*_c} $[V]$	**7563.28**	13100	43666.66	43666.66	0.99	**7558.37**
(5) S_{*_c} [VA]	**133333.33**	400000	133333.33	400000	0.8	**133164.65**
(6) $\boldsymbol{I}_{*\_c}$ [A]	$\mathbf{17.63\measuredangle -25.84^o}$	$17.62\measuredangle -25.84^o$	$3.05\measuredangle -25.84^o$	$5.28\measuredangle -25.84^o$	$0.8061\measuredangle -25.84^o$	$\mathbf{17.61\measuredangle -25.83^o}$
(7) $\Delta\boldsymbol{V}_{*\_\#}$ [V]	$491.52\measuredangle 54.16^o$	$851.34\measuredangle 54.16^o$	$2834.30\measuredangle 54.16^o$	$2832.82\measuredangle 54.16^o$	$0.0644\measuredangle -54.16^o$	NA
(8) $\boldsymbol{V}_{*\_e}$ [V]	$7861.18\measuredangle 2.90^o$	$13615.98\measuredangle 2.90^o$	$\mathbf{45384.40\measuredangle 2.90^o}$	$45383.48\measuredangle 2.90^o$	$1.029\measuredangle -2.90^o$	$\mathbf{45384.39\measuredangle 2.90^o}$
(9a) $R_{m\_\#}$ [Ω]	11616	11616	**387200**	387200	33.3333	**387200**
(10a) $\boldsymbol{X}_{m\_\#}$ [Ω]	$17424\measuredangle 90^o$	$17424\measuredangle 90^o$	$580800\measuredangle 90^o$	$580800\measuredangle 90^o$	$50\measuredangle 90^o$	NA
(11) $\boldsymbol{Z}_{m\_\#}$ [Ω]	$9665.09\measuredangle 33.69^o$	$9665.098\measuredangle 33.69^o$	$322169.87\measuredangle 33.69^o$	$322169.87\measuredangle 33.69^o$	$27.7350\measuredangle 33.69^o$	NA
(12) $\boldsymbol{I}_{m\_\#}$ [A]	$0.81\measuredangle -30.79^o$	$0.81\measuredangle -30.79^o$	$\mathbf{0.14\measuredangle -30.79^o}$	$0.24\measuredangle -30.79^o$	$0.0371\measuredangle -30.79^o$	$\mathbf{0.14\measuredangle -31.41^o}$
(13) $\boldsymbol{I}_{*\_e}$ [A]	$18.43\measuredangle -26.05^o$	$18.43\measuredangle -26.05^o$	$\mathbf{3.19\measuredangle -26.05^o}$	$5.52\measuredangle -26.05^o$	$0.8430\measuredangle -26.05^o$	$\mathbf{3.21\measuredangle -25.61^o}$
(14) P_{fe_t} [W]	5319.98	15959.94	**5319.58**	15958.75	0.0317	**5112.9**
(15) P_{cu_t} [W]	1504.76	4514.28	**1501.12**	4503.37	0.0090	**1562**
(16) η	94.61 %	94.61 %	94.61 %	94.61 %	94.64 %	94.46 %
(17) V_{Reg}	3.93 %	3.93 %	3.93 %	3.93 %	3.9393 %	4.01 %
(18) fp_e	0.87 ↓	0.87 ↓	0.87 ↓	0.87 ↓	0.87 ↓	0.87 ↓

It is worth to notice that certain values such as copper losses, efficiency, voltage regulation and input power factor are not directly obtained by means of the ATPDraw simulation. However, these values can be calculated by means of other results provided in the simulation. For example, copper losses (P_{cu_t}) can be obtained adding the power losses in each winding. Transformer efficiency is calculated with equation (16). Voltage regulation can be obtained using equation (17). Finally, input power factor is calculated using equation (18).

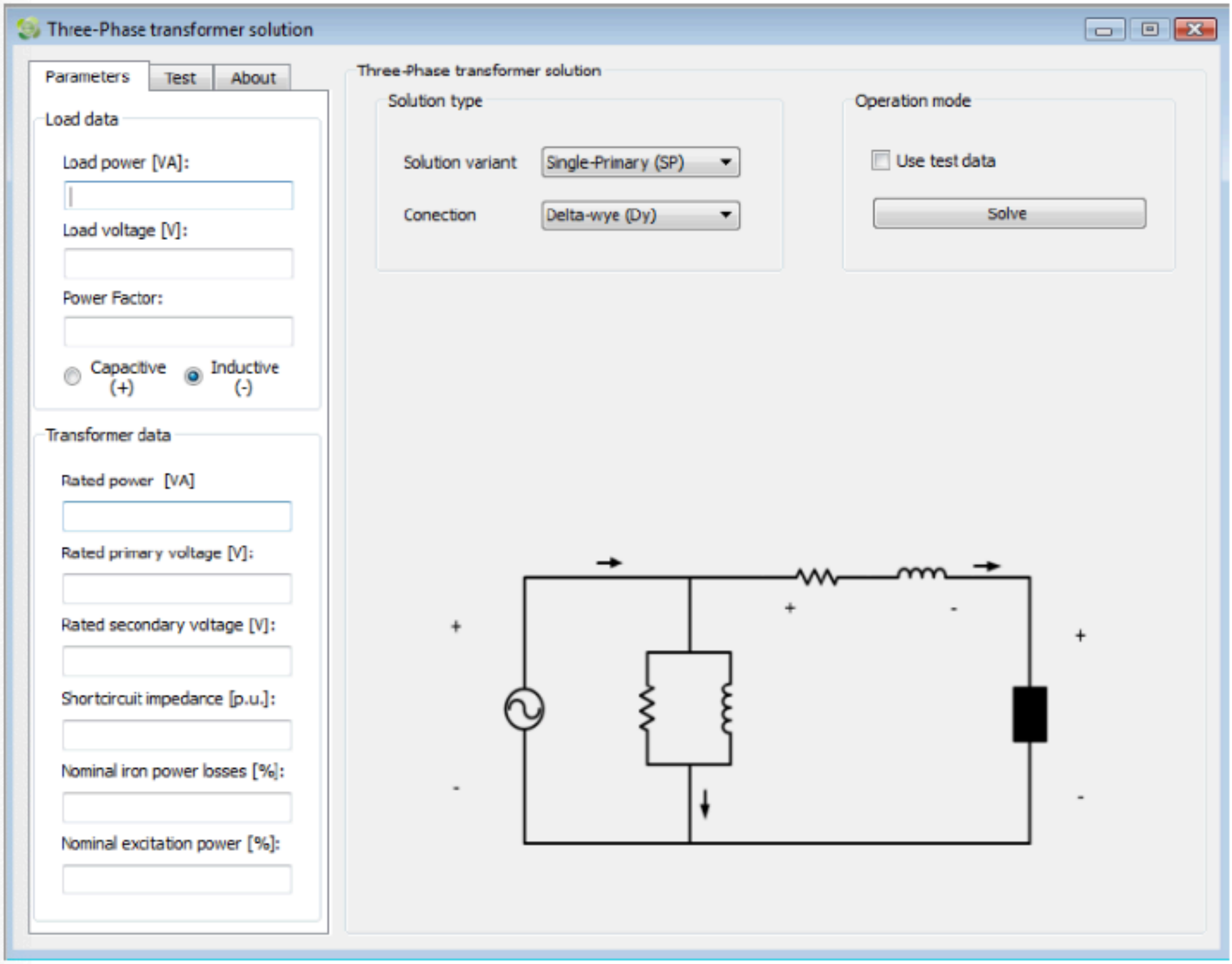
According to the results presented in Table 4 it can be verified that SS and TS (ΔY connections) have the following equivalences: first, in wye connections line current and phase current are the same; second, in wye connections the line-line voltage is the line-neutral voltage multiplied by $\sqrt{3}$; third, the three-phase power is three times the single-phase power. Also note that SP and TP (ΔY connections) have the following equivalences: first, in delta connections the line-line voltage is equal to the line-neutral voltage; second, in delta connections the line current is the phase current multiplied by $\sqrt{3}$; finally, the power calculated in TP can be obtained multiplying by 3 the power obtained in the SP variant.

Figure 7 shows the main window of the graphic interface implemented in Python to obtain the transformer parameters and variables with the proposed method. The five variants have been implemented in the graphic interface, this one presents the results in the transformer simplified model (see figure 4). The interface can be found in the follow web page:

https://github.com/IceMerman/TransformerSolution

In this interface the user can obtain the parameters and variables of three-phase transformers. In the main window the user indicates the load data and transformer data from the identification plate; also, the user can select method variant and transformer winding connection.

Figure 7
Graphic interface



○ Capacitive (+) ● Inductive (-)

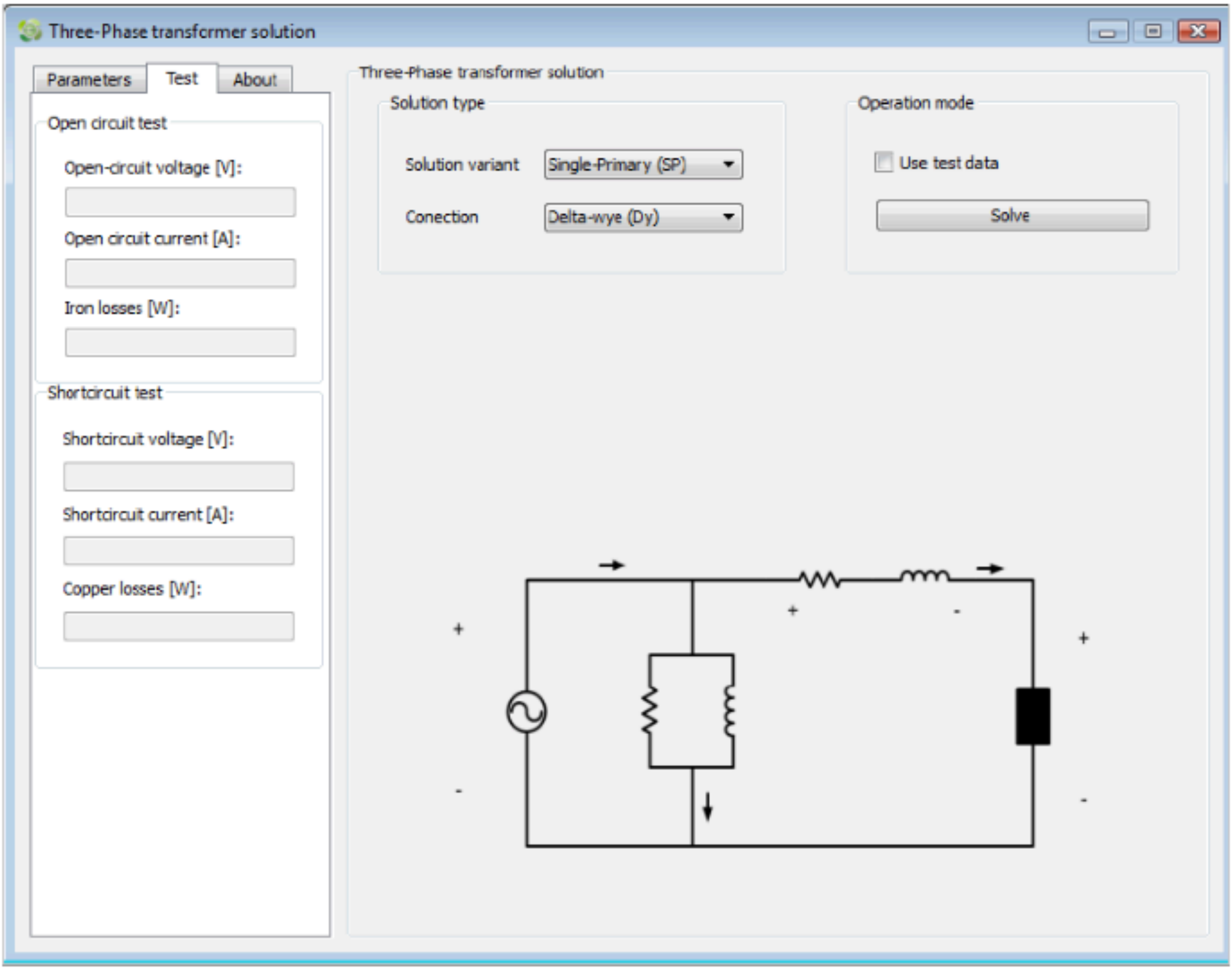
Solution variant: Single-Primary (SP)

Conection: Delta-wye (Dy)

☐ Use test data

Figure 8 shows the test data panel of the graphic interface. This panel is activated when the option is marked in the “operation mode” section. In this panel, the user can enter the transformer short-circuit and open-circuit test data for the calculation of short-circuit and magnetization impedances.

Figure 8
Test data panel of
the graphic interface

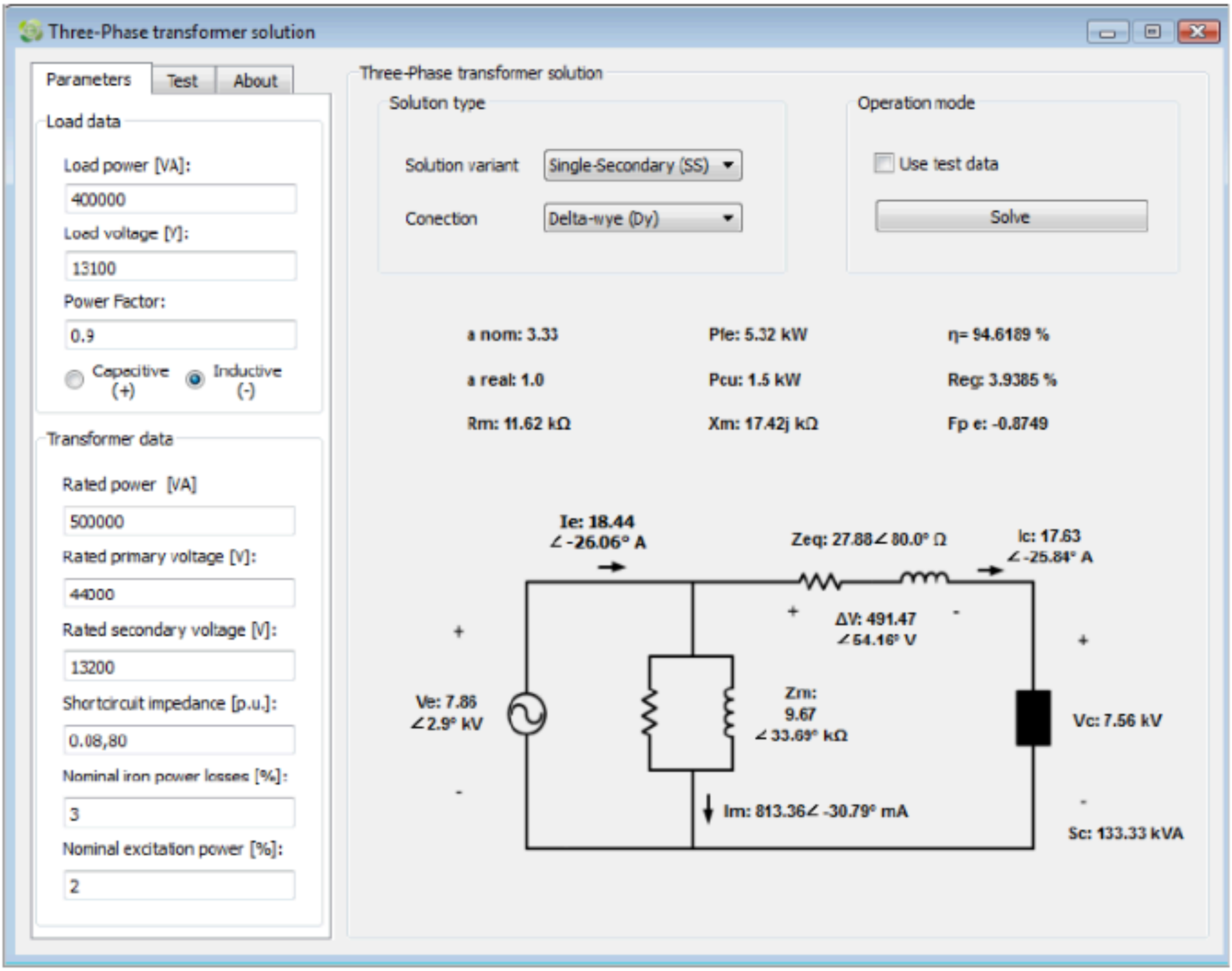


Solution variant: Single-Primary (SP)

Conection: Delta-wye (Dy)

Finally, the “solve” button is used for obtaining the calculation results. Figure 9 shows the results of the example solved in this paper for the SS variant.

Figure 9
Calculation results
for SS variant



Load data

Load power [VA]: 400000

Load voltage [V]: 13100

Power Factor: 0.9

○ Capacitive (+) ● Inductive (-)

Transformer data

Rated power [VA]: 500000

Rated primary voltage [V]: 44000

Rated secondary voltage [V]: 13200

Shortcircuit impedance [p.u.]: 0.08,80

Nominal iron power losses [%]: 3

Nominal excitation power [%]: 2

Three-Phase transformer solution

Soluton type

Solution variant: Single-Secondary (SS)

Conection: Delta-wye (Dy)

Operation mode

☐ Use test data

a nom: 3.33
Pfe: 5.32 kW
η= 94.6189 %
a real: 1.0
Pcu: 1.5 kW
Reg: 3.9385 %
Rm: 11.62 kΩ
Xm: 17.42j kΩ
Fp e: -0.8749

Ie: 18.44 ∠-26.06° A
Zeq: 27.88∠80.0° Ω
Ic: 17.63 ∠-25.84° A
ΔV: 491.47 ∠54.16° V
Ve: 7.86 ∠2.9° kV
Zm: 9.67 ∠33.69° kΩ
Vc: 7.56 kV
Im: 813.36∠-30.79° mA
Sc: 133.33 kVA

4. Conclusions

In this paper, a method with five variations to solve the equivalent circuit of three-phase transformers was proposed and explained. This method allows calculating the electric variables (voltages, currents, powers, efficiency, power factors and voltage regulation) of transformer according to load condition, transformer parameters and transformer test data. The proposed method uses the simplified transformer model and was validated by means of simulations in ATPDraw. Despite of the fact that ATPDraw uses a different transformer model, simulation results were equivalent.

Mathematical results and simulations demonstrate the equivalence between the five method variants. Therefore, it is possible to present a general method to find the transformer variables. The application of the method is useful for engineers in making technical and economic decisions regarding transformer selection and its related variables; as well as for students for the understanding of transformers' performance.

The results obtained with the graphical interface implemented in PYTHON were consistent with the simulations performed in ATPDraw. In this interface, students as well as engineers can obtain the transformer parameters and variables entering the load characteristics, the transformer data from the identification plate and the results of short-circuit and open-circuit tests.

Bibliographic references

Abdulsalam, S. G., Xu, W., & Dinavahi, V. (2005). Modelling and simulation of three-phase transformers for inrush current studies. *IEE Proceedings - Generation, Transmission and Distribution*, *152*(3), 328–333. https://doi.org/10.1049/ip-gtd:20041060

Ayasun, S., & Nwankpa, C. O. (2006). Transformer tests using MATLAB/Simulink and their integration into undergraduate electric machinery courses. *Computer Applications in Engineering Education*, *14*(2), 142–150. https://doi.org/10.1002/cae.20077

Celis-Montero, J. E., Castro-Aranda, F., & Martinez-Velasco, J. A. (2012). Modelo práctico del transformador de distribución trifásico para análisis de transitorios de baja frecuencia: Identificación de parámetros. *Ingeniare. Revista Chilena de Ingeniería*, *20*, 293–303.

Georgilakis, P. S., & Amoiralis, E. I. (2010). Distribution transformer cost evaluation methodology incorporating environmental cost. *IET Generation, Transmission & Distribution*, *4*(7), 861–872. https://doi.org/10.1049/iet-gtd.2009.0638

Hassan Hosseini, S. M., Vakilian, M., & Gharehpetian, G. (2008). Comparison of Transformer Detailed Models for Fast and Very Fast Transient Studies. *IEEE Transactions on Power Delivery*, *23*(2), 733–741. https://doi.org/10.1109/TPWRD.2008.915795

Huan, W., Xiangrui, S., & Guangzhe, J. (2012). The study of switch transformer medium voltage asynchronous motor soft-start device. *Power Electronics and Motion Control Conference (IPEMC), 2012 7th International*, *4*, 2422–2425. https://doi.org/10.1109/IPEMC.2012.6259235

Hurng-Liahng, J., Kuen-Der, W., Jinn-Chang, W., & Wen-Jung, C. (2008). A Three-Phase Four-Wire Power Filter Comprising a Three-Phase Three-Wire Active Power Filter and a Zig–Zag Transformer. *IEEE Transactions on Power Electronics*, *23*(1), 252–259. https://doi.org/10.1109/TPEL.2007.911779

Ionescu, G., Paltanea, G., & Paltanea, V. (2013). A method for minimization the harmonic distortions in three-phase inverter devices. *Advanced Topics in Electrical Engineering (ATEE), 2013 8th International Symposium On*, 1–4. https://doi.org/10.1109/ATEE.2013.6563380

Jiale, S., Jiao, Z., Song, G., & Kang, X. (2009). Algorithm to identify the excitation inductance of power transformer with wye-delta connection. *IET Electric Power Applications*, *3*(1), 1–7. https://doi.org/10.1049/iet-epa:20080037

Martínez-Velasco, J. A., & de León, F. (2011). Circuito equivalente de un transformador con regulación. *Ingeniare. Revista Chilena de Ingeniería*, *19*, 93–109.

Nagy, A., & Osama, M. (2010). Physics-Based High-Frequency Transformer Modeling by Finite Elements. *IEEE Transactions on Magnetics*, *46*(8), 3249–3252. https://doi.org/10.1109/TMAG.2010.2048017

Thompson, M., Miller, H., & Burger, J. (2008). Innovative power flow regulating tap-changer control installed on multiple phase-shifting transformers. *Transmission and Distribution Conference and Exposition, 2008. T&D. IEEE/PES*, 1–9. https://doi.org/10.1109/TDC.2008.4517270

Vahidi, B., Agheli, S. A., & Jazebi, S. (2012). Teaching short-circuit withstand test on power transformers to M.Sc. students and junior engineers using MATLAB-SIMULINK. *Computer Applications in Engineering Education*, *20*(3), 484–492. https://doi.org/10.1002/cae.20416

Vahidi, B., & Esmaeeli, E. (2013). MATLAB-SIMULINK-based simulation for digital differential relay protection of power transformer for educational purpose. *Computer Applications in Engineering Education*, *21*(3), 475–483. https://doi.org/10.1002/cae.20493

Varan, M., & Yurtsever, U. (2017). Multi-DGAS: A pattern based educational framework design for power transformers faults interpretation and comparative performance analysis. *Computer Applications in Engineering Education*. https://doi.org/10.1002/cae.21872

Xiao-pin, D., Yi-yi, Z., Yue, T., & Bin, L. (2014). A life cycle cost-effectiveness assessment model for power transformer selection based on grey correlation analysis. *Electricity Distribution (CICED), 2014 China International Conference On*, 1168–1173. https://doi.org/10.1109/CICED.2014.6991891

Yang, Y., Wang, Z. J., Cai, X., & Wang, Z. D. (2011). Improved lumped parameter model for transformer fast transient simulations. *IET Electric Power Applications*, *5*(6), 479–485. https://doi.org/10.1049/iet-epa.2010.0114

Yazdani-Asrami, M., Mirzaie, M., & Shayegani Akmal, A. A. (2010). Investigation on impact of current harmonic contents on the distribution transformer losses and remaining life. *Power and Energy (PECon), 2010 IEEE International Conference On*, 689–694. https://doi.org/10.1109/PECON.2010.5697668

---

[1] Master student. Department of Electrical Engineering. Research group of efficient energy management (GIMEL). University of Antioquia. jrobinson.ortiz@udea.edu.co

[2] Ph.D. in Electronics and Automation. Department of Electrical Engineering. Research group of efficient energy management (GIMEL). University of Antioquia. alvaro.jaramillod@udea.edu.co

[3] Ph.D. in Electronics. Department of Electrical Engineering. Research group of efficient energy management (GIMEL). University of Antioquia. nicolas.munoz@udea.edu.co

[4] Master student. Department of Electrical Engineering.Figure 6. Input data for ATPDraw simulation. Research group of efficient energy management (GIMEL). University of Antioquia. juane.sierra@udea.edu.co

[5] Ph.D. in Electrical Engineering. Department of Electrical Engineering. Research group of efficient energy management (GIMEL). University of Antioquia. jmaria.lopez@udea.edu.co

---

Revista ESPACIOS. ISSN 0798 1015
Vol. 40 (Nº 29) Year 2019

[Index]